13. V. N. Nikolaevskii, É. A. Bondarev, M. I. Mirkin, et al., Movement of Hydrocarbon Mixtures in a Porous Medium [in Russian], Nedra, Moscow (1968).
14. M. D. Rozenberg, S. A. Kundin, A. K. Kurbanov, et al., Filtration of a Gasified Liquid and Other Multicomponent Mixtures in Oil-Bearing Strata [in Russian], Nedra, Moscow (1969).

HELICAL FLOW OF A NONLINEARLY VISCOPLASTIC LIQUID IN AN ANNULAR CHANNEL

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This article presents analytical expressions describing the flow of a generalized nonlinearly viscoplastic liquid in a concentric annular channel under complex shear. The results of numerical calculations are analyzed.

The theoretical and experimental investigation of helical flow in pipes was dealt with by many authors of whom we mention [1-5]. Rivlin [1], e.g., obtained some general relations for the helical flow of a generalized non-Newtonian liquid in an annular channel. The experimental verification of the generalized regularity of flow under conditions of complex shear was carried out by Vinogradov et al. [2]. Coleman et al. [3] obtained general relations for the distribution of longitudinal and angular velocities in a channel and the flow rate of a generalized non-Newtonian liquid. However, numerical calculations with their aid were carried out for the first time by Prokunin et al. [4] for an exponential liquid. All these authors dealt with liquids that do not have a limit shear stress. As far as the few existing investigations dealt with viscoplastic liquids, only the qualitative aspect of the problem was studied, and this does not make it possible to carry out numerical calculations and their corresponding analysis. For instance, Myasnikov [5] obtained a phase diagram expressing qualitatively the nature of profiles of longitudinal and angular velocity of the helical flow of a Bingham—Shvedov liquid. The present authors obtained a closed system of equations in dimensionless form enabling them to calculate the principal characteristics of the helical flow of a nonlinearly viscoplastic liquid in an annular channel, both for the normal and the inverse hydraulic problem.

We examine laminar steady-state flow in a concentric annular channel formed by two long cylinders with radii a and b (a < b), with constant pressure gradient $-P = -\Delta p/l$, acting along the cylinder axis z. The inner cylinder rotates at constant angular velocity Ω_0. As rheological model we use the generalized model of Shul'man which is adequate for the rheological behavior of various paint and varnish compositions, pulps, foodstuffs, cement and clay suspensions, and a number of other non-Newtonian media:

$$\eta\dot{\gamma} = (\tau^{1/n} - \tau_0^{1/n})^m, \quad \tau > \tau_0,$$
$$\dot{\gamma} = 0, \quad \tau \leqslant \tau_0. \tag{1}$$
$$\dot{\gamma} = \sqrt{(u_r')^2 + (r\omega_r')^2}, \tag{2}$$

τ is the tangential stress intensity,

$$\tau = \sqrt{(\tau_{rz})^2 + (\tau_{r\theta})^2}. \tag{3}$$

Expressions (1)-(3) are written with a view to the axisymmetric nature of the helical flow. For the stress components τ_{rz} and $\tau_{r\theta}$ the following relations ensuing from the equations of equilibrium of an element of the liquid [1] are correct:

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$$\tau_{rz} = \frac{d}{r} - \frac{rP}{2}, \tag{4}$$

$$\tau_{r\theta} = \frac{s}{r^2}. \tag{5}$$

For the components of the deformation rate of the helical flow in the directions z and θ we have

$$u_r' = \frac{\tau_{rz}}{\tau}\dot{\gamma}, \tag{6}$$

$$\omega_r' = \frac{\tau_{r\theta}}{r\tau}\dot{\gamma}. \tag{7}$$

For the general case we assume that within the flow there is a core with the boundaries r_1 and r_2 ($r_1 < r_2$); by integrating Eqs. (6) and (7), on condition of adhesion to the channel walls and with a view to relations (1)-(5), we obtain for the velocity field of the flow the following system of equations in dimensionless form:

$$V(\xi) = \frac{\mathrm{Sen}^{m/n}}{2(1-k)\beta_0^{m/n}} \int_1^{\xi} \frac{\beta_1 - \zeta^2}{\zeta} \frac{f(\zeta)}{\beta(\zeta)} d\zeta \quad (\xi_2 \leqslant \xi \leqslant 1), \tag{8}$$

$$V(\xi) = \frac{\mathrm{Sen}^{m/n}}{2(1-k)\beta_0^{m/n}} \int_k^{\xi} \frac{\beta_1 - \zeta^2}{\zeta} \frac{f(\zeta)}{\beta(\zeta)} d\zeta \quad (k \leqslant \xi \leqslant \xi_1), \tag{9}$$

$$\Omega(\xi) = \frac{k\,\mathrm{Sen}^{m/n}}{2(1-k)\beta_0^{m/n}} \int_1^{\xi} \frac{\beta_2}{\zeta^3} \frac{f(\zeta)}{\beta(\zeta)} d\zeta \quad (\xi_2 \leqslant \xi \leqslant 1), \tag{10}$$

$$\Omega(\xi) = \frac{k\,\mathrm{Sen}^{m/n}}{2(1-k)\beta_0^{m/n}} \int_k^{\xi} \frac{\beta_2}{\zeta^3} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \frac{1}{\mathrm{Ro}} \quad (k \leqslant \xi \leqslant \xi_1), \tag{11}$$

where

$$f(\zeta) = [\beta(\zeta)^{1/n} - \beta_0^{1/n}]^m; \tag{12}$$

$$\beta(\zeta) = \sqrt{\left(\frac{\beta_1 - \zeta^2}{\zeta}\right)^2 + \frac{\beta_2^2}{\zeta^4}}\ . \tag{13}$$

On the boundaries ξ_1 and ξ_2 the following conditions apply:

$$V(\xi_1) = V(\xi_2), \quad \Omega(\xi_1) = \Omega(\xi_2). \tag{14}$$

Hence, with a view to (8)-(11) we obtain

$$\int_k^{\xi_1} \frac{\beta_1 - \zeta^2}{\zeta} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \int_{\xi_2}^{1} \frac{\beta_1 - \zeta^2}{\zeta} \frac{f(\zeta)}{\beta(\zeta)} d\zeta = 0, \tag{15}$$

$$\int_k^{\xi_1} \frac{\beta_2}{\zeta^3} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \int_{\xi_2}^{1} \frac{\beta_2}{\zeta^3} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \frac{2(1-k)\beta_0^{m/n}}{k\,\mathrm{Ro}\,\mathrm{Sen}^{m/n}} = 0. \tag{16}$$

The dimensionless boundaries of the core of the flow ξ_1 and ξ_2 are determined as the roots of the equation

$$f(\xi) = 0. \tag{17}$$

We substitute relations (12) and (13) into (17) and represent the equation for determining the boundaries of the core thus:

$$\xi^2(\xi^2 - \xi_-^2)(\xi^2 - \xi_+^2) + \beta_2^2 = 0, \tag{18}$$

where

$$\xi_- = \left(\sqrt{\beta_0^2 + 4\beta_1} - \beta_0\right)/2; \tag{19}$$

$$\xi_+ = \left(\sqrt{\beta_0^2 + 4\beta_1} + \beta_0\right)/2. \tag{20}$$

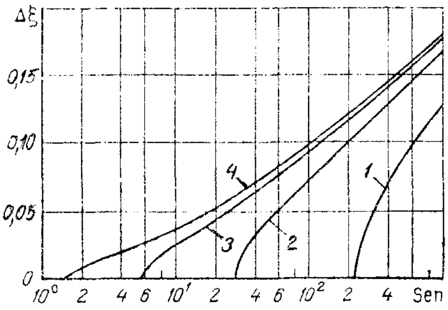

Fig. 1

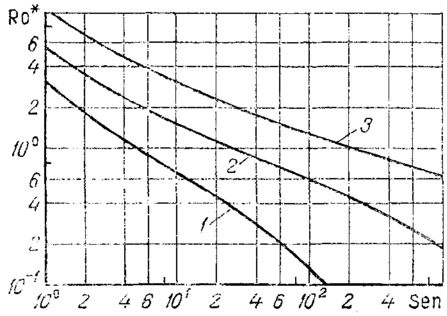

Fig. 2

Fig. 1. Dependence of the thickness of the core $\Delta\xi$ on the St. Venant number (Sen) for k = 0.6 and n = 3: 1) Ro = 1; 2) 2; 3) 4; 4) 8.

Fig. 2. Dependence of the critical Rossby number (Ro*) on Sen for k = 0.6: 1) n = 1; 2) 2; 3) 3.

In the general case the expression for the flow rate through the annular channel has the form

$$Q = 2\pi \int_a^b ru(r)\, dr. \tag{21}$$

After some transformations and the use of relations (8)-(11) we obtain from expression (21) that

$$\int_k^{\xi_1} \frac{(\beta_1 - \zeta^2)\zeta}{2} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \int_{\xi_2}^{1} \frac{(\beta_1 - \zeta^2)\zeta}{2} \frac{f(\zeta)}{\beta(\zeta)} d\zeta + \frac{(1-k)(1-k^2)\beta_0^{m/n}}{\mathrm{Sen}^{m/n}} = 0. \tag{22}$$

Thus we obtained the closed system of equations (15), (16), and (22) with respect to the unknowns β_0, β_1, β_2 with the specified parameters Sen, Ro, k, n, m. Knowing β_0, we can determine the pressure losses by known formulas in fractions of the dynamic shear stress τ_0:

$$\Delta p = \frac{2l}{\beta_0 b} \tau_0 \tag{23}$$

or via the velocity head

$$\Delta p = \lambda \frac{l}{2(b-a)} \frac{\rho W^2}{2}, \tag{24}$$

where

$$\lambda = \frac{8(1-k)\,\mathrm{Sen}}{\beta_0\,\mathrm{Re}}. \tag{25}$$

Having calculated Δp, we can find the torque applied to the inner cylinder:

$$M = \pi\beta_2 \Delta p b^3. \tag{26}$$

We point out that for m = n = 1 and the corresponding choice of the other dimensionless parameters, expression (15) corresponds in regard to accuracy to relation (2.15) obtained by Myasnikov [5].

The system of equations (15), (16), and (22) was solved numerically. In addition to calculating the unknowns β_0, β_1, and β_2, we also determined the velocity profile by dependences (8)-(11). These calculations fully confirmed Myasnikov's qualitative conclusions concerning the nature of the velocity profiles in complex shear and contained in [5]. That means that the following cases of helical flow are possible: without core; with a core within the flow and a characteristic protrusion on the profile of the longitudinal velocities; with a core in the flow without the characteristic protrusion; with a core adjacent to the outer cylinder.

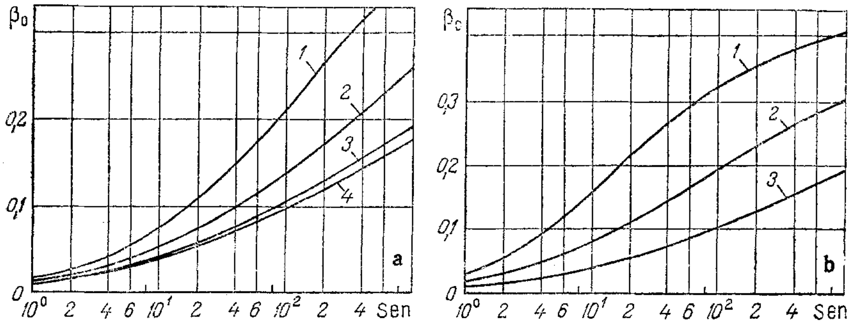

Fig. 3. Dependence of the parameter β_0 on Sen: a) for k = 0.6 and n = 3: 1) Ro = 0.06; 2) 0.2; 3) 1; 4) 10; b) for k = 0.6 and Ro = 1: 1) n = 1; 2) 2; 3) 3.

All this also applies to the flow of nonlinearly viscoplastic liquids. In the special case, when $\Omega_0 = 0$ and m = n, the obtained solution coincides completely with the solution presented by Bukhman et al. [6].

Let us examine some results of calculations carried out for m = n. The dependence of the thickness of the core $\Delta\xi = f(\text{Sen, Ro})$ on the criteria of similarity is shown in Fig. 1. It can be seen from an analysis of this dependence that for any Ro number the core becomes thinner when Sen is lower. The core also becomes thinner when Ro becomes smaller (with the same Sen). With increasing Sen the dependence of $\Delta\xi$ on Ro manifests itself more weakly. For instance, a change of Ro from 8 to 2 with Sen = 100 leads to a decrease of $\Delta\xi$ by about 23%, and with Sen = 1000 to a decrease of about 9%.

Figure 2 shows the dependence of the critical Rossby numbers at which the core vanishes, Ro* = f(Sen, n). With constant n, Ro* decreases with increasing Sen.

With increasing n (Sen = const), Ro* increases, and all the more rapidly, the larger Sen is.

The dependence $\beta_0 = f\,(\text{Sen, Ro})$ is presented in Fig. 3a. For any fixed Ro, β_0 also increases, i.e., the coefficient of hydraulic resistance λ decreases (see formula (25)). Reduced Ro with any fixed Sen (i.e., increased rotational speed of the cylinder with constant flow rate) also leads to increased β_0 and corresponding decrease of λ. For instance, a change of Ro from 1 to 0.06 with Sen = 100 leads to a decrease of λ to one half.

Figure 3b shows the dependence $\beta_0 = f(\text{Sen, n})$. With fixed Sen an increase of n leads to reduced β_0 and increased λ, and consequently to pressure loss due to friction.

A more thorough physical analysis can be carried out fairly simply by examining the effect of the actual parameters (Q, Ω_0, τ_0, η, a, b, etc.) contained in the dimensionless Ro and Sen numbers on the behavior of these criteria. For instance, an increase of the flow rate Q (with the other parameters unchanged) leads to a decrease of Sen and an increase of Ro. An analogous pattern is found when τ_0, Ω_0, etc. decrease. Calculations showed, in particular, that when the rotational speed is constant, increased flow rate entails a reduced moment applied to the inner cylinder.

Thus it is obvious that notation of a cylinder substantially affects the nature of the velocity profiles, the dimensions and position of the core, the magnitude of the moment and of the pressure losses when a nonlinearly viscoplastic liquid flows in an annular channel.

NOTATION

τ_{rz}, $\tau_{r\theta}$, stress components; d, s, integration constants; τ_0, limit shear stress; m, n, indices of nonlinearity of the rheological model; a, b, radii of the inner and outer cylinders, respectively; γ, deformation rate intensity; η, analog of plastic viscosity; ρ, density of the liquid; u, ω, running axial and angular velocities, respectively, of the flow; r_1, r_2, core boundaries; Ω_0, angular velocity of rotation of the inner cylinder; r, θ, z, cylindrical coordinates; Q, volume flow rate of the liquid; λ, coefficient of hydraulic resistance; $W = Q/\pi\,(b^2 - a^2)$, mean flow rate over the section; M, torque. Dimensionless parameters: $k = a/b$; $\xi = r/b$, coordinate; $V = u/W$, running flow velocity; $\Omega = \omega a/W$, angular velocity of the flow; ξ_1, ξ_2, core boundaries; $\Delta\xi = \xi_2 - \xi_1$, thickness of the core; $\beta_0 = 2\tau_0 l/\Delta pb$; $\beta_1 =$

$2dl/\Delta pb^2$; $\beta_2 = 2sl/\Delta pb^3$; $Sen = \tau_0[2(b - a)/\eta W]^{n/m}$, St. Venant number; $Re = \rho W^2[2(b - a)/\eta W]^{n/m}$, Reynolds number; $Ro = W/\Omega_0 a$, Rossby number.

LITERATURE CITED

1. R. S. Rivlin, "Solution of some problems in the exact theory of viscoelasticity," J. Rational Mech. Anal., No. 1, 179-188 (1956).
2. G. V. Vinogradov, A. A. Mamakov, and N. V. Tyabin, "Flow of anomalously viscous bodies under conditions of a state of complex stress," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mashinostr., No. 2, 65-69 (1960).
3. B. D. Coleman, H. Markovitz, and W. Noll, Viscosimetric Flows of Non-Newtonian. Theory and Experiment, New York (1966).
4. A. N. Prokunin, M. L. Fridman, and G. V. Vinogradov, "Helical flow of polymeric media," Mekh. Polim., No. 3, 497-505 (1971).
5. V. P. Myasnikov, "In complex shear," Prikl. Mekh. Tekh. Fiz., No. 5, 76-87 (1961).
6. U. A. Bukhman, V. I. Lipatov, A. I. Litvinov, B. I. Mitelman, and Z. P. Shulman, "Rheodynamics of nonlinear viscoplastic media," J. Non-Newtonian Fluid Mech., 10, 215-233 (1982).

UNSTEADY TWO-DIMENSIONAL FLOW OF A COMPRESSIBLE NON-NEWTONIAN FLUID IN A LONG ANNULAR CHANNEL CAUSED BY THE MOTION OF AN INSIDE PIPE

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We solve the unsteady, two-dimensional problem of the hydrodynamics of a compressible non-Newtonian fluid connected with the study of the flow in an annular channel caused by the motion of an inside pipe.

One of the complex operations in drilling is the lowering and raising of the column of drill pipes, which must be done regularly to replace the drill bit when it becomes dull. We note that in deep and superdeep drilling, the lowering and raising operations take up a large fraction of the total time and consist of the periodically repeated lowering (raising) of the column of drill pipes by a length of one drill-pipe stand (about 12-36 m). After this, the following stand is attached (disconnected), and the next lowering (raising) is carried out. These operations lead to the formation, in the drilling mud, of strong, periodically repeated disturbances, in the channel of the borehole, which, after propagating along the channel, and being reflected from its ends, superimposed, and damped, produce dynamic loads on the walls of the well, which often lead to different complications during drilling. Analogous effects arise during lowering of the column of the casings.

A number of theoretical studies devoted to this question are known [1, 2]. However, because of the complexity of the problem being considered, these studies completely or partially neglect such important factors as the unsteadiness of the phenomenon, the compressibility of the fluid, the non-Newtonian properties of the fluid, and the two-dimensionality of the flow picture.

We attempt to eliminate the indicated shortcomings.

We consider the following problem. We have a long vertical pipe of length L, radius R_2 with a closed end (Fig. 1), filled with a non-Newtonian compressible fluid with specific density ρ, having known rheology. Inside the pipe there is lowered another pipe, coaxial with it, having length $L_1 < L$, radius R_1 with an end that is closed by means of a valve, through which we can pump a fluid in a single direction with volume flow rate $q_1(t)$. The upper end of the annular pipe is open and communicates with the atmosphere.

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